Oxford Cambridge and RSA

GCE

Mathematics A

H240/01: Pure Mathematics

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for A Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless the question specifically asks for another level of accuracy. Follow through should be used so that only one mark is lost for each distinct accuracy error.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			**DR** $(5x - 2)(2x + 1) > 0$	**M1**	**1.1a**	Factorise 3 term quadratic	Need $a = 10$, and either $b = 1$ or $c = -2$ when expanded Or solve using a valid method If using the formula allow one sign slip
			$x = -\frac{1}{2}$, $x = \frac{2}{5}$	**A1**	**2.1**	Obtain both correct roots	Could be implied by the two values appearing in an incorrect inequality **SC** allow B1 in place of M1A1 if roots are given but with no evidence of solving the quadratic SC B1 includes $(x + \frac{1}{2})(x - \frac{2}{5})$ unless division by 10 seen prior to factorisation
			$x < -\frac{1}{2}$, $x > \frac{2}{5}$	**M1**	**1.1a**	Select outside region	For their two distinct roots Allow M1A0 for $x \le -\frac{1}{2}$, $x \ge \frac{2}{5}$ Allow M1A0 for $\frac{2}{5} < x < -\frac{1}{2}$ or with $\le$
				A1	**2.5**	Obtain correct inequalities	Any correct notation, including set notation, but A0 if linked by ‘and’ **SC** Allow B2 for answer only (B1 for sight of correct roots and B1 for correct inequality)
				[4]			

Question			Answer	Marks	AO	Guidance	
2	(a)	(i)	Show A in third quadrant, with length of 8 and relevant angle marked on given axes	B1 [1]	1.2	Allow any correct angle	Condone A being located by correct **i** and **j** components instead of length and angle – could be stated as a coordinate or values marked on the axes
		(ii)	$x = 8\cos 240^{o} = -4$ $y = 8\sin 240^{o} = -4\sqrt{3}$	M1	1.1a	Attempt both components from magnitude of 8 and an angle	Could use 60^{o} (no need to consider whether positive or negative for this mark) Allow M1 for $8\cos\theta$ and $8\sin\theta$ attempted Condone a value for θ that may not be consistent with their diagram Max of M1 only, if A incorrect on diagram
			A is $-4\mathbf{i} - 4\sqrt{3}\,\mathbf{j}$	A1	1.1	Obtain one correct component	Condone eg $x = -4$ for $-4\mathbf{i}$
				A1 [3]	1.1	Obtain fully correct position vector	Allow 6.93, or better, for $4\sqrt{3}$ A0 if coordinate or column vector
	(b)		area $= 0.5 \times 8 \times 6 \times \sin 120^{o}$	M1	3.1a	Attempt area of triangle, using correct formula	M0 if 240^{o} used Allow plausible angle ie 30^{o}, 60^{o}, 120^{o}, 150^{o} Allow other incorrect angles as long as explicit on their diagram Allow multi-step methods as long as fully correct method
			$= 12\sqrt{3}$	A1 [2]	1.1	Obtain $12\sqrt{3}$	Must be exact www eg M1A0 for $12\sqrt{3}$ from A in second quadrant M1A0 for $12\sqrt{3}$ from using 60^{o} without justification that $\sin 120^{o} = \sin 60^{o}$

Question			Answer	Marks	AO	Guidance	
	(c)		$6\mathbf{i} - (-4\mathbf{i} - 4\sqrt{3}\,\mathbf{j})$	M1	3.1a	Attempt 6**i** – (their OA)	Allow BOD for $6\mathbf{i} - - 4\mathbf{i} - 4\sqrt{3}\,\mathbf{j}$, even if final answer is not commensurate with 'invisible brackets'
			C is $10\mathbf{i} + 4\sqrt{3}\,\mathbf{j}$	A1	1.1	Obtain $10\mathbf{i} + 4\sqrt{3}\,\mathbf{j}$	Allow 6.93, or better, for $4\sqrt{3}$
				[2]			**SC** B1 for $2\mathbf{i} - 4\sqrt{3}\,\mathbf{j}$ or $-10\mathbf{i} - 4\sqrt{3}\,\mathbf{j}$ ie a valid parallelogram having misinterpreted *OABC*

Question			Answer	Marks	AO	Guidance	
3	(a)		$k = 3$	B1 [1]	2.2a	State 3	B0 for $k \geq 3$ Allow B1 for $x \geq 3$, as this implies $k = 3$
	(b)		f(5) = – 13	M1	1.2	Attempt f(5)	Could be implied by –13 Could be part of algebraic attempt at ff(x), with $x = 5$ used, but does not need to be evaluated for M1
			–13 is not in domain so f(–13), and hence ff(5), is not defined	A1 [2]	1.1	Correct conclusion	Allow equiv, such as 'not possible' **SC** Allow A1 for f(– 13) = 239
	(c)		$(x - 3)^2 - 17 = x$ $x^2 - 7x - 8 = 0$ $x = 8$, $x = -1$	M1	1.1a	Equate and attempt to solve	**BC** Equate and produce at least one root, not necessarily correct for their equation Could be implied by sight of 8, or 8 and –1, even if equation not seen
			Obtain at least $x = 8$	A1	1.1	If second root is given, it must also be correct	www, eg $x = 8$ given as only root from $(x - 8)(x - 1)$ is M1A0
			$x = -1$ is not valid as $x \geq 3$, so $x = 8$	A1 [3]	2.3	Obtain $x = 8$ only, having discarded $x = -1$, with a reason such as 'not in the domain' or 'less than 3'	Must be using $k = 3$; if referring to 'less than k' then 3 must have been seen in part **(a)** Must see some indication that the other root would have been –1, eg a factor of $(x + 1)$ or a numerical quadratic formula not fully evaluated
	(d)		f(x) and $f^{-1}(x)$ are reflections in the line $y = x$ so the point of intersection must be on $y = x$	B1 [1]	1.2	Correct description	Sufficient to see f(x) and $f^{-1}(x)$ intersect on $y = x$ **or** reference to reflections in $y = x$

Question			Answer	Marks	AO	Guidance	
4	(a)		Identify AP with $a = 16000$ and $d = 1200$	B1	3.1b	Seen or implied	Could be implied by use in u_n, even if $n \neq 10$, or by use in S_n
			$u_{10} = 16000 + 9 \times 1200 =$ £26,800	B1	3.3	Obtain £26,800	Units required Answer only of 26,800 would be B1B0, as AP implied but no units
				[2]			
	(b)		$S_N = 0.5N(32000 + (N - 1)1200)$	M1	3.4	Attempt S_N of AP, with their a and d	FT their a and d from part **(a)**
			$600N^2 + 15400N - 500000 = 0$	A1	3.1a	Equate sum of AP to 500000 and rearrange to any correct 3 term quadratic	Allow =, or any inequality sign
			$N = 18.8$ (and possibly $N = -44.4$)	A1	1.1	At least correct positive root soi	**BC** Condone 18.7 (ie truncated not rounded) Sight of 19 implies A1, even if 18.8 never seen If other root given then must be correct
			19 years	B1FT	3.2a	Conclude with 19 (years) FT on their positive non-integer root being rounded up	Allow just 19, rather than 19 years B0 for ≥ 19, or equivalent in words the FT is just on their incorrect root (which must have come from an attempt at the sum of an AP, but could follow M0), and not any other aspect of the question Must see a non-integer value first to get the B1FT
				[4]			

Question			Answer	Marks	AO	Guidance	
	(c)		'not realistic' or 'unlikely to be realistic' along with a reason such as: Sam is unlikely to stay in the same role for that long Unlikely that salary will increase by same amount as may be an upper limit on annual salary eg 'pay cap', 'level off', 'plateau' Could reach a point where the company cannot afford to pay that salary Unlikely that salary will increase by same amount as likely to be a percentage increase	**B1** **[1]**	**3.5a**	Identify that model is not (very) realistic, with a sensible, specific reason which refers why the increase is unlikely to remain constant	Could use other language such as 'model may not hold', or 'model may not be valid' Could also identify that salary might increase by more eg Sam might get a promotion, be paid a bonus or be paid more due to inflation B0 for statements that do not pertain to this model, eg death, retirement or changing companies (allow B1 BOD for changing jobs as this could refer to a different role within the same company) If a correct reason has been given, then ignore other incorrect or irrelevant reasons, unless directly contradictory

Question			Answer	Marks	AO	Guidance	
5	(a)		$3x^2 - 6xy - 3x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$	M1*	1.1a	Attempt implicit differentiation	Either of the two $\frac{dy}{dx}$ terms correct, allowing sign errors
				A1	2.1	Correct derivative www	Condone no '= 0' on RHS Condone $\frac{dy}{dx} = \ldots$ as long as not used
			$3x^2 - 6xy + (2y - 3x^2)\frac{dy}{dx} = 0$ OR $-3x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 6xy - 3x^2$	M1d*	1.1a	Attempt to make $\frac{dy}{dx}$ the subject	**Either** collect like terms on each side **or** take out a common factor of $\frac{dy}{dx}$ Must have two terms involving $\frac{dy}{dx}$ and two terms without $\frac{dy}{dx}$
			$(2y - 3x^2)\frac{dy}{dx} = 6xy - 3x^2$ $\frac{dy}{dx} = \frac{6xy - 3x^2}{2y - 3x^2}$ **A.G.**	A1 **[4]**	2.1	Obtain correct $\frac{dy}{dx}$	Obtain given answer having collected like terms on either side and taken out a common factor (possibly with both steps being done in one go)
	(b)		$\frac{dy}{dx} = 9$	B1	1.1	Obtain gradient of 9	Could be implied by $-\frac{1}{9}$
			$m' = -\frac{1}{9}$	B1FT	1.1	Correct gradient of normal	FT their m
			$y - 2 = -\frac{1}{9}(x - 1)$	M1	1.1a	Attempt equation of normal	Using (1, 2) and their normal gradient (M0 if using gradient of tangent) Gradient must be numerical
			$x + 9y = 19$	A1 **[4]**	1.1	Obtain correct three term equation	Any correct equiv

Question			Answer	Marks	AO	Guidance	
6			$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$	**B1**	**1.1**	Correct expansion of $(x + h)^3$	Seen at any point Must have numerical coefficients not ${}^3C_1$ Condone 1 as a coefficient Could use δx oe instead of h Allow unsimplified ie like terms not collected
			$f(x+h) - f(x) =$ $\{2(x+h)^3 + 3(x+h)\} - \{2x^3 + 3x\}$ $= 2(x^3 + 3x^2h + 3xh^2 + h^3) + 3(x+h)$ $-2x^3 - 3x$	**M1**	**2.1**	Attempt to simplify $f(x + h) - f(x)$	If considering $2x^3$ and $3x$ separately then both must be considered for the M1 Could follow B0 but $f(x + h)$ must be a 4 term cubic Allow BOD for $\ldots - 2x^3 + 3x$
			$= 6x^2h + 6xh^2 + 2h^3 + 3h$	**A1**	**2.1**	Correct 4 term expression for $f(x + h) - f(x)$ www	Either one expression or two separate expressions
			$\dfrac{f(x+h) - f(x)}{h} =$ $\dfrac{6x^2h + 6xh^2 + 2h^3 + 3h}{h}$	**M1**	**2.5**	Attempt $\dfrac{f(x+h) - f(x)}{h}$	f must be in terms of the given function and not just a statement of the general definition $f(x + h)$ does not need to be expanded Allow even if $f(x + h)$ is now incorrect If considering $2x^3$ and $3x$ separately then both must be considered for the M1 Allow BOD for $\ldots - 2x^3 + 3x$
			$6x^2 + 6xh + 2h^2 + 3$	**A1**	**1.1**	Obtain correct expression www	
			$f'(x) = \lim_{h \to 0}(6x^2 + 6xh + 2h^2 + 3)$ $= 6x^2 + 3$	**A1**	**2.4**	Complete proof by considering limit as $h \to 0$	Must see 'lim', '$h \to 0$', and $f'(x)$ Dep on previous 5 marks being awarded
				[6]			**NB** Starting with $6x^2 + 3$ will get no credit in the entire question as not $f(x)$

Question			Answer	Marks	AO	Guidance	
7			**DR** GP, with $a = 15$, $r = 0.6$	**B1**	**3.1a**	Identify GP; correct a and r soi	Stated or implied by use in equation
			$S_\infty = \dfrac{15}{1-0.6}$	**B1**	**1.1a**	Correct S_∞, with their a and r	Must be using correct formula Allow $a = 25$, even if not stated explicitly before formula is used Allow $a = 15$, $r = 0.6$ and $\dfrac{a}{1-r} = 37.5$ to imply B1 B0 for 37.5 with no evidence
			$S_N = \dfrac{15(1-0.6^N)}{1-0.6}$	**B1**	**1.1a**	Correct S_N, with their a and r	Must be using correct formula Allow $a = 25$, even if not stated explicitly before formula is used
			$37.5 - 37.5(1 - 0.6^N) < 10^{-4}$ $37.5 \times 0.6^N < 10^{-4}$	**M1**	**3.1a**	Link $S_\infty - S_N$ to 10^{-4} and attempt to rearrange	As far as $p \times 0.6^N < q$ (q possibly 2 terms) Condone either '=' or any inequality sign M0 for eg $15 \times 0.6^N = 9^N$ or $1 - 0.6^N = 0.4^N$
			$0.6^N < 2.67 \times 10^{-6}$	**A1**	**1.1**	Correct equation in useable form	Any linking sign If using logs on 37.5×0.6^N then the product must be dealt with correctly to get both this A1 and the following M1

Question			Answer	Marks	AO	Guidance	
			$N > \log_{0.6}(2.67 \times 10^{-6})$	**M1**	**2.1**	Use logs to solve equation	Either take logs on both sides (consistent base), drop power and rearrange, or take $\log_{0.6}$ on RHS (could be base other than 0.6 if error when manipulating indices) Any linking sign, including an inequality sign that does not change direction
			$N > 25.125\ldots$ hence $N = 26$ **[8]**	**A1** **A1**	**1.1** **2.2a**	Obtain 25.1 / 25 / 26 Obtain $N = 26$ only (or eg N is 26) www	Any sign No evidence of use of logs – award B1 instead of M1A1 (and can still get final A1) A0 if inequality eg $N \geq 26$ A0 if it comes from an incorrect inequality eg $N < 25.125\ldots$ unless recovered by testing at least one relevant integer value If solving an equation then must test at least one integer value to justify N If either or both of the second and third B marks are not awarded for lack of **DR** then all other marks are available Answer only is 0/8 T&I could get some credit depending what equations are shown, but question requires both **DR** and an algebraic method so a final answer of 26 will not get credit

Question			Answer	Marks	AO	Guidance	
8	(a)		$\frac{dx}{dt} = k\sqrt{x}$	**B1***	**3.3**	Set up a correct differential equation	Allow – k
			– 0.0032 = $k\sqrt{0.64}$ so k = – 0.004 hence $\frac{dx}{dt} = -0.004\sqrt{x}$ **A.G.**	**B1d*** **[2]**	**3.3**	Obtain correct differential equation www	Or k = 0.004 from – k Must use –0.0032 when finding k B0 if k = 0.004, even if then comment about 'decreasing' and sign changed
	(b)		$\int -0.004\,dt = \int x^{-\frac{1}{2}}dx$	**M1***	**1.1a**	Separate variables and attempt integration	Condone dx, dt and/or integral sign not being explicit Could instead invert both sides of equation Increase by 1 in both of their powers
			$-0.004t = 2x^{\frac{1}{2}} + c$	**A1**	**1.1**	Correct integral – could still be in terms of k	Condone no + c Any correct equation eg $t = -500\sqrt{x} + c$
			– 0.004 × 100 = 2$\sqrt{0.64}$ + c c = –2	**M1d***	**3.4**	Use t = 100, x = 0.64 to find c	Substitute given values into their general solution and attempt c NB check method carefully for their equation and the position of their c
			$2\sqrt{x}$ = 2 – 0.004t $x = (1 - 0.002t)^2$	**A1** **[4]**	**1.1**	Correct equation	Any equiv as long as x in terms of t ISW if correct equation subsequently spoilt
	(c)		when x = 0, t = 500	**M1**	**1.1**	Use x = 0 to attempt a value for t	Must be using x = 0 in their particular solution, with a non-zero value for c
			so tank will be empty after 500 seconds	**A1** **[2]**	**3.4**	Units needed	Must come from using a correct equation Allow 8 mins 20 secs, $8.\dot{3}$ mins or 8.33 mins

Question			Answer	Marks	AO	Guidance	
9	(a)		$R^2 = 9 + 49$	M1	1.1a	Attempt correct process to find R	
			$R\cos\alpha = 3$, $R\sin\alpha = 7$ hence $\tan\alpha = \frac{7}{3}$	M1	1.1a	Attempt correct process to find $\tan\alpha$ (or equiv with $\sin\alpha$ or $\cos\alpha$)	M0 for $\tan\alpha = \frac{3}{7}$ Allow M1 even if then evaluated in degrees
			$\sqrt{58}\cos(3x - 1.17)$	A1 [3]	1.1	Obtain $\sqrt{58}\cos(3x - 1.17)$	Allow $R = 7.62$, or better α must be in radians If R and α are correct then no need to see them substituted back into the expression
	(b)		Stretch in the y direction by sf $\sqrt{58}$	B1FT	1.1	Follow through their R (numerical or just 'R') Given at any point in the sequence of transformations	Allow BOD if no 'scale factor' or equiv ie B1 for 'stretch in y –direction by $\sqrt{58}$ ' Must be 'parallel to y-axis', 'in y direction', 'x-axis invariant' or equiv, so B0 for 'along / in / on / to y-axis', 'parallel to y' etc
			Translation in the x direction by 1.17 Stretch in the x direction by sf $\frac{1}{3}$	M1	3.1a	Translation by ± their α and stretch by (sf) 3 or $\frac{1}{3}$, in either order, both in the x direction	Allow informal language for this mark eg 'shift', 'move', 'compression', 'squash' Allow translation by $\pm\frac{1}{3}$(their α)
				A1FT	1.1	Translation by their α (numerical, inc in degrees, or just 'α')	soi to be in the positive x-direction Must use correct language (see B1FT)
				A1 [4]	2.5	Stretch by sf $\frac{1}{3}$	A0A1 is possible For A1A1 stretch must follow translation, unless using $\frac{1}{3}$(their α) Must use correct language (see B1FT) Must mention 'scale factor', 'factor' or 'sf'

Question			Answer	Marks	AO	Guidance	
	(c)		greatest value is √58	B1FT	3.1a	FT their R	R must be numerical Allow no method shown
			when x = 0.389	B1	1.1	Obtain 0.389	Must be in radians 'Determine' so some method needed eg 3x – 1.17 = 0 oe (minimum of $x = \frac{1.17}{3}$) Allow 0.39
				[2]			
	(d)		least value is –√58	B1FT	3.1a	FT their R	R must be numerical Allow no method shown
			when x = 1.44	B1	1.1	Obtain 1.44	Must be in radians 'Determine' so some method needed eg 3x – 1.17 = π, or equiv
				[2]			
10	(a)		$\frac{1}{2}\times 6^2 \times \theta$	B1	1.2	Correct area of sector soi	Could be part of attempt at area of segment Allow unsimplified, inc $\pi \times 6^2 \times \frac{\theta}{2\pi}$
			$\frac{1}{2}\times 6^2 \times (\theta - \sin\theta) = 7.2$	M1	1.1	Attempt area of segment and equate to 7.2	Any equivalent method eg sector area = triangle area + 7.2 Correct formula for area of a triangle Area of sector must be $\left(\frac{1}{2}\right)\times 6^2 \times \theta$
			$\theta - \sin\theta = \frac{7.2}{18} = 0.4$ $\theta = 0.4 + \sin\theta$ **AG**	A1	2.1	Rearrange to obtain given answer	At least one line of working needed after equating to 7.2
				[3]			

Question			Answer	Marks	AO	Guidance	
	(b)		$F'(1.2) = \cos(1.2) = 0.362$	B1*	1.1	Correct $F'(1.2)$	Allow 0.36
			$\lvert F'(1.2)\rvert < 1$ so iteration will converge	B1d*	2.4	Identify correct condition	Must be $\lvert F'(1.2)\rvert < 1$ or $-1 < F'(1.2) < 1$ oe $F'(1.2) < 1$ is B0 as condition is incomplete, but condone $0 < F'(1.2) < 1$ oe in words
				[2]			
	(c)		1.3320	B1	1.1a	Correct first iterate	Allow 1.332
			1.3716, 1.3802, 1.3819, 1.3822, 1.3823…	M1	1.1	Attempt correct process	At least 3 iterations in total Must be given to at least 4 sf
			hence $\theta = 1.382$	A1	2.2a	Obtain $\theta = 1.382$ (must be 4sf)	Following at least 2 iterations that are 1.382 to 4sf A0 if given as $\theta_k = \ldots$ Allow recovery from an incorrect interim value (but B0M1A1 if first iterate is wrong)
				[3]			
	(d)		$1.3815 - 0.4 - \sin 1.3815 = -0.000637$ $1.3825 - 0.4 - \sin 1.3825 = 0.000175$	M1	2.1	Attempt relevant values either side of their root, using relevant θs	Allow other valid methods as long as they have one value either side of the root (1.382284…) and both round to 1.382, to 4sf Could use $\sin\theta + 0.4 = \theta$ and compare inequality signs
				A1	2.1	Obtain both correct values	For their θ values At least 1 sf Allow truncating not rounding
			$f(1.3815) < 0$ and $f(1.3825) > 0$ by sign change $1.3815 < \theta < 1.3825$ so must 1.382 correct to 4sf	B1	2.4	Must refer to sign change	Must be fully correct to award B1
				[3]			

Question			Answer	Marks	AO	Guidance	
11	(a)		$\int 1.\ln(x-4)\,dx$ so $u = \ln(x-4)$ and $v' = 1$	**M1**	**1.1a**	Attempt integration by parts, with correct parts	u and v' correctly allocated and correct formula used M0 if $v = x - 4$ from $v' = 1$
			$x\ln\lvert x-4\rvert - \int \frac{x}{x-4}\,dx$	**A1**	**1.1**	Correct expression	Allow brackets not modulus Allow $x \times \frac{1}{x-4}$, even if subsequently spoilt
			$\int \frac{x}{x-4}\,dx = \int 1 + \frac{4}{x-4}\,dx$	**M1**	**3.1a**	Attempt to deal with improper fraction	Allow sign error ie $1 - \frac{4}{x-4}$ Could use substitution of $u = x - 4$ but must get as far as a proper fraction (ie $1 \pm 4u^{-1}$) Do not need to actually integrate for M1
			$= x + 4\ln\lvert x-4\rvert$	**A1**	**1.1**	Correct integration of fraction	Allow brackets not modulus Using a substitution gives $x - 4 + 4\ln\lvert x-4\rvert$; must be in terms of x and not u for A1
			$\int \ln(x-4)\,dx$ $= x\ln\lvert x-4\rvert - x - 4\ln\lvert x-4\rvert + c$ $= (x-4)\ln\lvert x-4\rvert - x + c$ **A.G.**	**A1**	**2.4**	Show given answer with no errors seen	Modulus required in final answer, as well as $+ c$ Can go from penultimate line in MS to given answer with no further detail needed Answer from using substitution will need to justify changing c eg $c + 4$ is a constant hence c' is also a constant
				[5]			**NB** differentiating given answer is 0/5

Question			Answer	Marks	AO		Guidance
							OR B1 – use substitution of $v = x - 4$, with $\frac{dv}{dx} = 1$ seen, to obtain $\int \ln v\,dv$ if B0 as it is not explicit then next 4 marks are still available M1 – attempt integration by parts on $\int \ln v\,dv$, using correct parts and correct formula A1 – obtain $v\ln\lvert v\rvert - v$ (allow no modulus) A1 – obtain $(x-4)\ln\lvert x-4\rvert - (x-4)$ A1 – obtain given answer, including modulus sign, and with justification for their c becoming c'
	(b)		State $x = 4$	**B1** **[1]**	**2.2a**		Must be an equation
	(c)		$\left\lvert\int_5^7 \ln(x-4)dx\right\rvert + \left\lvert\int_{4.5}^5 \ln(x-4)dx\right\rvert$	**M1**	**3.1a**	Attempt sum of absolute areas	Or integral $\int_5^7 \ln(x-4)dx - \int_{4.5}^5 \ln(x-4)dx$
			$(3\ln 3 - 2) + (\frac{1}{2}\ln\frac{1}{2} + \frac{1}{2})$	**A1**	**1.1**	Correct expression for area	Any unsimplified equiv
			$3\ln 3 - 2 - \frac{1}{2}\ln 2 + \frac{1}{2}$	**M1**	**3.1a**	Attempt to rearrange to required form	Use $\ln\frac{1}{2} = -\ln 2$ and gather like terms Could follow M0 Allow M1 (implied) for 3ln3 + 0.5ln2 – 2.5, even if –0.5ln0.5 not seen first
			$3\ln 3 - \frac{1}{2}\ln 2 - \frac{3}{2}$	**A1**	**1.1**	Obtain $3\ln 3 - \frac{1}{2}\ln 2 - \frac{3}{2}$	Or $3\ln 3 - 0.5\ln 2 - 1.5$

Question			Answer	Marks	AO	Guidance	
				[4]			
12	(a)		$u = 3x^2$, $y = a^u$ $u' = kx$, $y' = a^u\ln a$	M1	1.1a	Attempt use of chain rule, with $y' = a^u\ln a$	Correct use of chain rule, with a^u correctly differentiated No credit for just stating $\frac{d}{dx}\left(a^x\right) = a^x \ln a$ unless clearly used in a correct chain rule
			$\frac{dy}{dx} = 6xa^u \ln a$	A1	2.1	Use chain rule to obtain correct derivative	Product of their u' and y' May still be in terms of x and u
			$\frac{dy}{dx} = 6xa^{3x^2} \ln a$ **A.G.**	A1	2.4	Obtain correct derivative	Now fully in terms of x
				[3]			**OR** M1 – attempt differentiation of $\ln y = 3x^2\ln a$ A1 – obtain $\frac{1}{y}\frac{dy}{dx} = 6x\ln a$ A1 – obtain correct derivative **A.G.** **OR** M1 – attempt differentiation of $y = e^{(3\ln a)x^2}$ A1 – obtain $\frac{dy}{dx} = (6x\ln a)e^{(3\ln a)x^2}$ A1 – obtain correct derivative **A.G.**

Question			Answer	Marks	AO	Guidance	
	(b)		when $x = 1$, $m = 6a^3\ln a$	B1	3.1a	Correct gradient of tangent soi	
			$y - a^3 = 6a^3\ln a(x - 1)$ $0 - a^3 = 6a^3\ln a\,(\frac{1}{2} - 1)$	M1*	1.1a	Use $(\frac{1}{2}, 0)$ in attempt at equation of tangent through $(1, a^3)$, or vice versa	OR $\frac{0 - a^3}{\frac{1}{2} - 1} = 6a^3\ln a$ M0 if gradient still in terms of x Allow BOD if something other than $x = 1$ was used to find the gradient
			$a^3 = 3a^3\ln a$ $a^3(3\ln a - 1) = 0$ $a = e^{\frac{1}{3}}$	M1d*	1.1a	Attempt to find a	Must go as far as attempting a value for a Condone cancelling by a^3 rather than factorising
				A1	1.1	Obtain correct value for a	Any equivalent exact form eg $a = \sqrt[3]{e}$
				[4]			
	(c)		$u = 6x\ln a$, $v = a^{3x^2}$	M1*	3.1a	Attempt use of product rule	On given $\frac{dy}{dx}$, with both parts a function of x If using parts as $u = 6xa^{3x^2}$ and $v = \ln a$, then must use product rule properly on u (but condone $\ln a$ differentiating to $\frac{1}{a}$)
			$\frac{d^2y}{dx^2} =$	A1	2.1	At least one term correct	
			$(6\ln a)(a^{3x^2}) + (6x\ln a)(6xa^{3x^2}\ln a)$ $= a^{3x^2}(6\ln a)(1 + 6x^2\ln a)$	A1	2.1	Fully correct second derivative	

Question			Answer	Marks	AO	Guidance	
			$\ln a > 0$ for $a > 1$ $a^{f(x)} > 0$ for all a and x $6x^2 \geq 0$, so $1 + 6x^2\ln a \geq 1$	**M1d***	**2.3**	Consider sign of each term – must consider each component of each term	Possibly factorised, or possibly considered term by term Domains not needed for the M1 Allow BOD if > not ≥
			$\frac{d^2y}{dx^2} > 0$ for all x hence curve is always convex	**A1** **[5]**	**2.2a**	Correct working only	Second derivative must be correct Domains must be seen Inequality signs must be correct throughout
			OR $\frac{dy}{dx} = 2xe^{x^2}$	**M1***		Attempt use of product rule	Substitute their a into $\frac{dy}{dx}$ and attempt to differentiate
			$\frac{d^2y}{dx^2} = 2e^{x^2} + 4x^2e^{x^2}$	**A1FT**		At least one term correct, FT their a¸ any a	
				A1FT		Fully correct second derivative, FT their a as long as of form e^k	Expect $6ke^{3kx^2} + 36k^2x^2e^{3kx^2}$
			$e^{x^2} > 0$ for all x, so $2e^{x^2} > 0$ $x^2 \geq 0$ for all x, so $4x^2e^{x^2} \geq 0$ hence $2e^{x^2} + 4x^2e^{x^2} > 0$	**M1d***		Consider sign of each term – must consider each component of each term	Possibly factorised, or possibly considered term by term Domains not needed for the M1 Allow BOD if > not ≥
			$\frac{d^2y}{dx^2} > 0$ for all x hence curve is always convex	**A1**		Correct working only	Second derivative must be correct Domains must be seen Inequality signs must be correct throughout

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